EVAPORATION OF MODERATELY COARSE DROPS OF BINARY SOLUTIONS IN A GAS MIXTURE

E. I. Alekhin and Yu. I. Yalamov

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A study is made of the evaporation of moderately coarse drops of binary solutions in a multicomponent mixture of gases. The results are compared with experimental data.

The rate of condensational growth or evaporation of moderately coarse aerosol particles depends to a significant extent on the evaporation coefficient α_m. The literature contains conflicting information on the value of this coefficient for water. Some studies [1, 2] give very low (of the order of 0.01) values for α_m, while others [3, 4] give values of the order of 1. A detailed analysis was made in [5] of many of the methods used to determine the evaporation coefficient.

In order to determine this coefficient, it is necessary to solve the problem of the evaporation of a moderately coarse aerosol particle. This problem has thus far been solved only through the use of methods which indirectly account for the effect of the Knudsen layer on the rate of evaporation [6-8]. It is interesting to attempt to solve this problem using exact boundary conditions obtained by the methods of the kinetic theory of gases from the solution of the system of Boltzmann equations [9].

We will examine a spherical drop of a concentrated solution of radius R. The drop is suspended in a ternary gas mixture. The first two components of the mixture have the same composition as the substances comprising the drop. We will use λ to designate the mean free path of the gas molecules. The ratio λ/R is referred to as the Knudsen number Kn. Drops for which $0.01 < \mathrm{Kn} \le 0.3$ are considered to be moderately coarse.

We will assume that both components of the liquid forming the drop undergo a phase transformation on the surface. We will further assume that constant concentrations of vapors of the volatile components $c_{1\infty}$. $c_{2\infty}$ and a constant mixture temperature T_∞ are maintained a large distance from the drop. Under the given conditions, the drop evaporation process can generally be considered quasisteady [10].

If the gradients of the temperature of the gas mixture and the concentrations of the components are small at distances of the order of the drop size, then the process of quasisteady evaporation can be described by the following system of linear differential equations:

$$\begin{gathered}
\eta^{(e)}\nabla^2\mathbf{v}^{(e)} = \nabla p^{(e)},\\
\operatorname{div}\mathbf{v}^{(e)} = 0,\\
\nabla^2 c_1^{(e)} = 0,\\
\nabla^2 c_2^{(e)} = 0,\\
\nabla^2 T^{(e)} = 0,\\
\eta^{(i)}\nabla^2\mathbf{v}^{(i)} = \nabla p^{(i)},\\
\operatorname{div}\mathbf{v}^{(i)} = 0,\\
\nabla^2 c_1^{(i)} = 0,\\
\nabla^2 T^{(i)} = 0.
\end{gathered} \tag{1}$$

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The superscripts e and i denote that the given physical quantity belongs to the regions outside and inside the drop, respectively; In Eq. (1), $v^{(i)}$ and $v^{(e)}$ are the mean-mass velocities of the liquid and gas mixtures; $T^{(i)}$ and $T^{(e)}$ are temperatures; $\eta^{(i)}$ and $\eta^{(e)}$ are the viscosities of the liquid and gas, respectively; $p^{(e)}$ and $p^{(i)}$ are the pressures outside and inside the drop. The relative concentration of the k-th component of the gas is determined by the relation $c_k^{(e)}=n_k^{(e)}/n^{(e)}$, where $n^{(e)}=n_1^{(e)}+n_2^{(e)}+n_3^{(e)}$. Inside the drop, $c_k^{(i)}=n_k^{(i)}m_k/\rho^{(i)}$. Here, $n_k^{(e)}$, $n_k^{(i)}$ are the concentrations of molecules of the k-th component inside and outside the drop, respectively; m_k is the mass of a molecule of the k-th component, $\rho^{(i)}=n_1^{(i)}m_1+n_2^{(i)}m_2$.

In accordance with the assumptions we have made, the following conditions will hold at large distances from the drop:

$$c_1^{(e)}|_{r\to\infty}=c_{1\infty},\quad c_2^{(e)}|_{r\to\infty}=c_{2\infty},$$
$$T^{(e)}|_{r\to\infty}=T_\infty,\quad p^{(e)}|_{r\to\infty}=p_\infty. \tag{2}$$

With allowance for conditions (2), the solution of system (1) appears as follows in cylindrical coordinates [11]:

$$v_r^{(e)}=\frac{A}{r^2},\quad c_1^{(e)}=\frac{B_1}{r},\quad c_2^{(e)}=\frac{B_2}{r},\quad T^{(e)}=\frac{C}{r}, \tag{3}$$
$$p^{(e)}=p_\infty,\quad v_r^{(i)}=v_{r0}^{(i)},\quad c_1^{(i)}=c_{10}^{(i)},\quad T^{(i)}=T_0^{(i)}.$$

To determine the constants in (3), it is necessary to assign boundary conditions on the surface of the drop. At r = R we will have the following:

1. The mass conservation laws for the first and second components of the liquid

$$n_k^{(e)}v_r^{(e)}-\frac{n^{(e)2}}{\rho^{(e)}}\sum_{j=1}^{2}D_{kj}m_j\left(\frac{dc_j^{(e)}}{dr}\right)-\frac{D_k^{(T)}}{m_k}\left(\frac{dT^{(e)}}{dr}\right)=$$
$$=\frac{I_k}{4\pi R^2m_k},\quad k=1,\ 2. \tag{4}$$

2. The condition of nonflow of molecules of the third component

$$n_3^{(e)}v_r^{(e)}-\frac{n^{(e)2}}{\rho^{(e)}}\sum_{j=1}^{2}D_{3j}m_j\left(\frac{dc_j^{(e)}}{dr}\right)-\frac{D_3^{(T)}}{m_3}\left(\frac{dT^{(e)}}{dr}\right)=0. \tag{5}$$

3. The condition of continuity of the heat flow

$$-\varkappa^{(e)}\left(\frac{dT^{(e)}}{dr}\right)+\varkappa^{(i)}\left(\frac{dT^{(i)}}{dr}\right)=-(L_1m_1n_1^{(e)}+L_2m_2n_2^{(e)})v_r^{(e)}+$$
$$+\frac{n^{(e)2}}{\rho^{(e)}}\sum_{j=1}^{2}m_j(L_1m_1D_{1j}+L_2m_2D_{2j})\left(\frac{dc_j^{(e)}}{dr}\right)+(L_1D_1^{(T)}+ \tag{6}$$
$$+L_2D_2^{(T)})\left(\frac{dT^{(e)}}{dr}\right),$$

where $\varkappa^{(e)}$ and $\varkappa^{(i)}$ are the thermal conductivity of the drop and gas, respectively; L_k is the heat of vaporization of the k-th component; I_k is the rate of change in the mass of the k-th component in the drop.

4. The condition of continuity of the normal component of the tensor of the viscous stresses

$$-p^{(e)} + 2\eta^{(e)}\left(\frac{dv_r^{(e)}}{dr}\right) - 2\frac{\sigma}{R} = -p^{(i)} + 2\eta^{(i)}\left(\frac{dv_r^{(i)}}{dr}\right), \tag{7}$$

where σ is the surface tension of the substance of the drop.

It is also necessary to assign boundary conditions for the concentration of the volatile components and the temperature of the gas with allowance for discontinuities.

5. The boundary condition for the temperature of the gas mixture

$$T^{(e)} - T^{(i)} = K_T^{(T)}\left(\frac{dT^{(e)}}{dr}\right) + T_\infty \sum_{j=1}^{2} K_T^{j(n)}\left(\frac{dc_j^{(e)}}{dr}\right), \tag{8}$$

where $K_T^{(T)}$, $K_T^{j(n)}$ are the temperature discontinuity coefficients. Temperature and concentration discontinuity coefficients were calculated in [9].

6. The boundary condition for the concentration of the k-th volatile component

$$\begin{gathered} c_k^{(e)} = \Phi_k(T_\infty,\ R,\ c_1^{(i)}) + \frac{\partial\Phi_k}{\partial T}(T^{(i)} - T_\infty) + \\ + \frac{K_{kn}^{(T)}}{T_\infty}\left(\frac{dT^{(e)}}{dr}\right) + \sum_{j=1}^{2} K_{kn}^{j(n)}\left(\frac{dc_j^{(e)}}{dr}\right), \end{gathered} \tag{9}$$

where $\Phi_k(T_\infty,\ R,\ c_1^{(i)})$ is the concentration of the saturated vapors of the k-th component and $K_{kn}^{(T)}$, $K_{kn}^{j(n)}$ are the concentration discontinuity coefficients.

With allowance for conditions (4)-(9), the expression for the rate of change of the mass of the k-th component can be written in the form:

$$\begin{gathered} \frac{dM_k}{dt} = -4\pi m_k \sum_{j=1}^{2} B_j \left\{ D_{kj} - \frac{n_k^{(e)}}{n_3^{(e)}} D_{3j} - \right. \\ \left. -\theta_j\left(\frac{D_k^{(T)}}{m_k} - \frac{n_k^{(e)}}{n_3^{(e)}}\frac{D_3^{(T)}}{m_3}\right)\right\}(\Phi_j(T_\infty,\ R,\ c_1^{(i)}) - c_{j\infty}), \end{gathered} \tag{10}$$

where

$$\theta_j = \frac{n^{(e)^2}}{\rho^{(e)}} m_j \frac{\sum_{j=1}^{2}\left\{\left(\frac{D_3^{(T)}}{m_3}\right)^{(2-k)} \sum_{l=1}^{2} (-1)^{l+1} L_1 m_1 n_1^{2-k} D_{1j}^{k-1}\right\}}{\varkappa^{(e)} + \sum_{j=1}^{2} L_j\left(D_k^{(T)} + \frac{n_k^{(e)} m_k}{n_3^{(e)} m_3} D_3^{(T)}\right)};$$

$$B_1 = \frac{a_2^{(2)} - a_2^{(1)}}{a_1^{(1)}\, a_2^{(2)} - a_2^{(2)}\, a_1^{(2)}}, \quad B_2 = \frac{a_1^{(1)} - a_1^{(2)}}{a_1^{(1)}\, a_2^{(2)} - a_1^{(2)}\, a_2^{(1)}};$$

$$a_j^{(j)} = 1 + \frac{K_{jn}^{j(n)}}{R} + \frac{\partial \Phi_j}{\partial T} \frac{K_T^{j(n)}}{R} T_\infty - \theta_j \left\{ \frac{K_{jn}^{(T)}}{RT_\infty} - \frac{\partial \Phi_j}{\partial T} \left(1 + \frac{K_T^{(T)}}{R} \right) \right\}; \tag{11}$$

$$a_k^{(j)} = \frac{K_{kn}^{j(n)}}{R} + \frac{\partial \Phi_k}{\partial T} \frac{K_T^{j(n)}}{R} T_\infty - \theta_j \left\{ \frac{K_{kn}^{(T)}}{RT_\infty} - \frac{\partial \Phi_j}{\partial T} \left(1 + \frac{K_T^{(T)}}{R} \right) \right\}.$$

We find the temperature of the surface of the liquid drop from the following equation:

$$T^{(i)} - T_\infty = \sum_{j=1}^{2} B_j \left\{ -\theta_j \left(1 + \frac{K_T^{(T)}}{R} \right) + T_\infty \frac{K_T^{j(n)}}{R} \right\}.$$

As a special case, we can use Eq. (9) to obtain an expression for the rate of evaporation of a drop of binary solutions in which one of the components is nonvolatile. To do this, we set $c_2^{(e)}=0$, $\Phi_2=0$, $D_2^{(e)}=0$, $D_{2j}=0$ in Eqs. (9)-(10). In this case, outside the drop we will have a binary mixture of the carrier gas and vapors of the solvent. Thus, the generalized diffusion coefficients D_{ij} can be replaced by the usual interdiffusion coefficients D_{ij}:

$$D_{11} = \frac{m_3}{m_1} \tilde{D}_{13}, \quad D_{31} = -\tilde{D}_{31}, \quad D_{13} = D_{33} = 0.$$

Ignoring thermodiffusion phenomena, we obtain the following expression for the rate of change of the mass of the drop:

$$R \frac{dR}{dt} = -\frac{\tilde{D}_{13} m_1 n^{(e)}}{\rho_1^{(i)} c_2^{(e)}} B, \tag{12}$$

where

$$B = \frac{\Phi_1 (T_\infty,\ R,\ c_1^{(i)}) - c_{1\infty}}{1 + \theta \frac{\partial \Phi_1}{\partial T} \left(1 + \frac{K_T^{(T)}}{R} \right) + c_{1\infty} \frac{K_n^{(n)}}{R} - \frac{\partial \Phi_1}{\partial T} T_\infty \frac{K_T^{(n)}}{R} - \frac{\theta c_{1\infty}}{T_\infty} \frac{K_n^{(T)}}{R}},$$

$$\theta = \frac{L \tilde{D}_{13}}{\varkappa^{(e)}} \frac{m_1 n^{(e)}}{c_3^{(e)}}.$$

Equation (12) differs from the formulas obtained in [3, 4, 6-8] in the presence of the cross terms in the temperature and concentration jumps. We can use Eq. (12) to obtain an expression which describes the evaporation of drops of pure liquids. In the limiting case of coarse drops (Kn < 0.01), Eq. (12) becomes the well-known expressions in [12] for the rate of drop evaporation.

Unfortunately, there is currently no experimental data on the evaporation of moderately coarse drops of binary solutions of volatile liquids. We will therefore use data on the evaporation of drops of salt solutions [3] and compare it with the rate of evaporation calculated from Eq. (12).

The authors of [3] conducted three series of experiments. In the first series, drops were grown on condensation nuclei with a corrected radius R_0 = 0.18 μm. In the second series, R_0 = 0.26 μm, while R_0 = 0.40 μm in the third series. Figure 1 shows results of the calculations

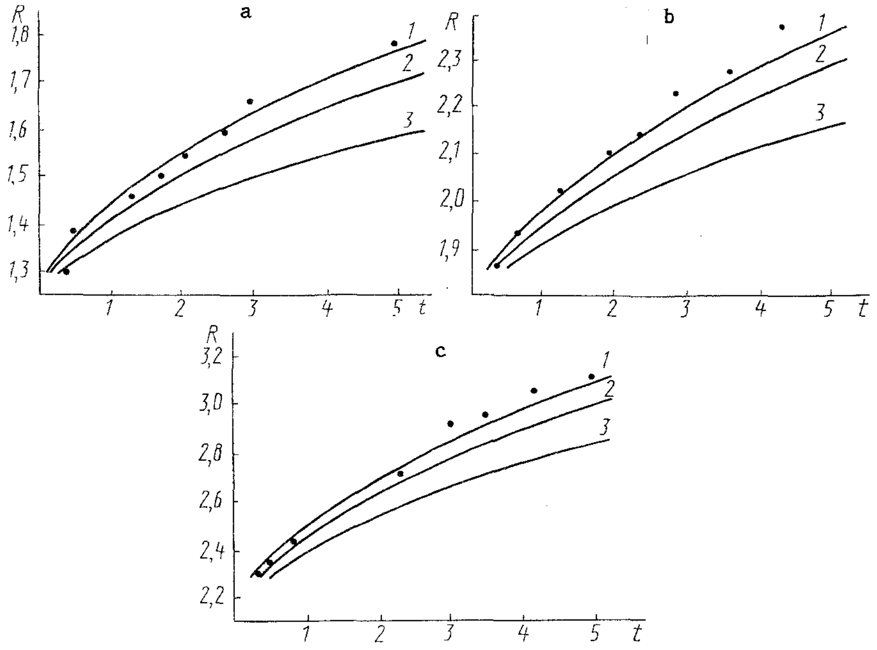

Fig. 1. Dependence of the corrected radius on time for different initial values of drop size: a) R_0 = 0.18 μm; b) 0.26; c) 0.40; 1) α_m = 1; 2) 0.1; 3 – 0.03. R, μm; t, C.

(solid curves) and experimental data. The rate of drop evaporation was calculated for three values of the evaporation coefficient: α_m = 1, 0.1, and 0.03. It should be noted that the following expressions (with our notation) were used in [3] to calculate drop growth rate:

$$R\frac{dR}{dt}=\frac{\tilde{D}_{13}m_1n^{(e)}}{\rho_1^{(i)}}\,\frac{\Phi_1(T_\infty,\ R,\ c_1^{(i)})-c_{1\infty}}{1+\theta+\dfrac{4\tilde{D}_{13}}{\bar{V}R\alpha_m}+\theta\dfrac{\varkappa^{(e)}}{mR\alpha_T}},$$

where $\bar{V}$ is the mean velocity of the gas molecules; m is a coefficient which characterizes the thermal contact of the drop with air (it is connected with the temperature jump). The quantity $4\tilde{D}_{13}/\bar{V}R\alpha_m$, characterizing the effect of the Knudsen layer evaporation rate, differs roughly 70% from the value calculated by the Loyalki method [9] for α_m = 1. The total contribution of crossover effects connected with the discontinuity coefficients $K_T^{(n)}$ and $K_n^{(T)}$ is 15% of the contribution of the coefficient $K_n^{(n)}$.

Comparison of the theoretical results obtained using rigorous kinetic theory and the empirical data supports the conclusion in [3] that the evaporation coefficient of water lies within the interval 0.1-1. It is not possible to determine α_m more precisely due to the large scatter of the experimental data.

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DOUBLE EXPLOSION ABOVE A HEATED SURFACE

V. A. Andrushchenko and
M. V. Meshcheryakov

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The Navier—Stokes equations are used to numerically solve a problem on the interaction of a pair of spherical waves with one another and an underlying rigid surface covered by a layer of heated gas. A study is made of complex processes involving the irregular collision of primary and secondary shock waves, their interaction with thermal discontinuities, the accompanying formation of precursors, suspended shocks, vortices, etc.

The interaction of shock waves (SW) has recently been a subject of intensive study in astrophysics, the theory of explosion, plasma physics, etc. The authors of [1, 2] examined the collision of normal and oblique shocks, while a unidimensional problem on the direct collision of two plane shock waves was solved numerically [3]. The study [4] focused on the interaction of spherical SW's in a double explosion in which point explosions occurred at the same point in space but at slightly different times. There has been little attention given to the more complicated effects which arise in the frontal collision of spherical SW's, most of the studies having been experimental investigations conducted in laboratory [5] and full-scale [6] experiments. Such collisions have also been examined in connection with experiments studying the interaction of laser sparks [7, 8]. Similar numerical problems were studied in [5, 7, 8] and were solved in two-dimensional formulations in [9].

In the present study, we examine the more complex problem of a double explosion above a hard surface in the presence of a thermal layer (TL) on the surface. Here, along with the frontal collision of spherical SW's, there is an interaction between the SW's and the surface, another interaction between the SW's and the TL, and interactions with reflected and secondary SW's.

1. We solved the problem of the interaction of two laser-induced explosions (the parameters of which are close to the parameters examined in [7, 8]). The explosions occur simultaneously on a single vertical line above a solid surface covered by a layer of heated gas formed either as a result of radiation from the explosions or as a result of independent heating (see [10, 11]).

As the mathematical model of the given physical process, we choose the system of Navier—Stokes equations for a compressible heat-conducting gas in cylindrical coordinates (r, z) for the axisymmetric case. In dimensionless variables, this system has the form:

$$\frac{d\mathbf{v}}{dt} = \frac{1}{\rho}\left\{ -\nabla p + \frac{1}{\mathrm{Re}}\left| \nabla(\mu\nabla\cdot\mathbf{v}) + \frac{1}{3}\nabla(\mu\,\mathrm{div}\,\mathbf{v}) \right]\right\},$$

$$\frac{dT}{dt} = -(\gamma-1)T\,\mathrm{div}\,\mathbf{v} + \frac{\gamma}{\rho\,\mathrm{RePr}}\nabla(k\nabla T), \qquad (1)$$

$$\frac{dp}{dt} = -\gamma p\,\mathrm{div}\,\mathbf{v} + \frac{\gamma}{\mathrm{Re\,Pr}}\nabla(k\nabla T), \qquad p = \rho T, \quad \frac{d}{dt} \equiv \frac{\partial}{\partial t} + (\mathbf{v}\cdot\nabla).$$

Department of Theoretical Problems, Academy of Sciences of the USSR, Moscow.

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